

Fuzzy detour ss-interior vertices and fuzzy detour ss-boundary vertices

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Abstract-In this paper some properties of Fuzzy detour ss-eccentric vertices is obtained. Fuzzy detour ss-boundary vertices and fuzzy detour ss-interior vertices in a fuzzy graph are introduced. we obtain some relationship between fuzzy cut vertex and fuzzy detour ss- interior vertex. Also some properties of fuzzy detour ss boundary vertex, fuzzy detour ss- interior vertex and complete vertex are discussed. Fuzzy detour ss-interior vertex and fuzzy detour ss- boundary vertex of a fuzzy tree are characterized .

Index Terms-detour ss-interior, fuzzy detour ss- boundary vertex.

1. INTRODUCTION

In 1975 Azriel Rosenfeld introduced the theory of fuzzy graph.. Yeh and Bang developed many concepts of fuzzy graph. μ - distance in fuzzy graph theory was introduced by Rosenfeld. Also he defined many graph theoretical concepts including paths, connectedness, cut nodes ,clique, bridge, trees and forest. Bhutani and Rosenfeld introduced strong arcs and strong path and g-distance in fuzzy graph. The concepts of eccentricity and centre based on μ -distance in fuzzy graphs is developed by Sunitha and Vijayakumar. Fuzzy planer graph was introduced by Abdul Jabbar et al. Tarashankar defined Interval valued fuzzy planar graph and also he defined interval valued fuzzy dual graph. Akram introduced the concepts of bipolar fuzzy graph and also he defined interval valued fuzzy number. Linda and Sunitha introduced the concepts of g-peripheral nodes, g-boundary vertices, and g-interior vertices based on g-distance. The concepts of fuzzy detour μ - distance was introduced by Nagoorgani and Umamaheswari. Linda and Sunitha introduced fuzzy detour g-distance. The author further defined g-boundary vertices and fuzzy detour g-interior vertices in fuzzy graph. The idea of strong sum distance was introduced by Minitom and M.S.Sunitha. Based on this concepts, they introduced boundary and interior vertices in a fuzzy graph. In this paper, we introduce the concepts of fuzzy detour ss boundary and interior vertices of a fuzzy graph based on detour ss-distance.

Section 2 contains preliminaries and in section 3 contains detour ss-distance of a fuzzy graph. Also detour ss-eccentricity, detour ss-radius, detour ss-diameter, detour ss-central vertices

and detour ss-peripheral vertices are given. It has definition of fuzzy ss- detour graph. Section 4 ,Fuzzy detour ss-periphery and fuzzy detour ss eccentric sub graph are defined. Some Properties of

detour ss-eccentric vertex, and detour ss-peripheral vertex are discussed. In section 5, fuzzy detour ss-boundary vertices of a fuzzy graph is defined also some properties of detour ss-boundary vertices with cut vertices are discussed. In section 6, fuzzy detour ss-interior vertices of a fuzzy graph is defined also some properties of detour ss- interior vertices of a fuzzy graph with detour ss-boundary vertices and cut vertices are discussed.

2. PRELIMINARIES

Let V be any non empty finite set . Then fuzzy graph is denoted by $G:(V, \sigma, \mu)$ where σ is a fuzzy subset of V and μ is a fuzzy relation on σ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for every $u, v \in V$. It is assumed that μ is reflexive and symmetric. In all the examples σ is chosen suitably. Underlying crisp graph is denoted by $G^* : (\sigma^*, \mu^*)$ where σ^* is the set of all $u \in G$ such that $\sigma(u) > 0$ and μ^* is the set of all $(u, v) \in V \times V$ such that $\mu(u, v) > 0$. It is assumed that $\sigma^* = V$. A fuzzy graph $H:(V, \tau, \gamma)$ is called Partial fuzzy graph of $G:(V, \sigma, \mu)$ if $\tau(u) \leq \sigma(u) \forall u \in \tau^*$ and $\gamma(u, v) = \mu(u, v) \forall (u, v) \in \gamma^*$ and if in addition $\tau^* = \sigma^*$, then H is called a spanning fuzzy sub graph of G . An arc in $G:(V, \sigma, \mu)$ is called weakest arc if it has least membership value. A path P of length n is a sequence of distinct nodes u_0, u_1, \dots, u_n such that $\mu(u_{i-1}, u_i) > 0 \ i=1, 2, 3, \dots, n$ and strength of the path is the

degree of membership of a weakest arc in the path. If $u_0 = u_n$ and $n \geq 3$ then P is called a cycle and cycle P is called fuzzy cycle if it contains more than one weakest arc. A fuzzy graph $G:(V, \sigma, \mu)$ is a complete fuzzy graph if $\mu(u, v) = \sigma(u) \wedge \sigma(v), \forall u, v \in \sigma^*$. Let $G:(V, \sigma, \mu)$ be a fuzzy graph. The strength of connectedness between two nodes x and y is defined as the maximum of the strength of all paths between x and y and it is denoted by $CONNG(x, y)$. A fuzzy graph $G:(V, \sigma, \mu)$ is called connected if for every x, y in σ^* , $CONNG(x, y) > 0$. Through this, we assume that G is connected. An arc of a fuzzy graph $G:(V, \sigma, \mu)$ is called strong if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted. An u - v path P is called strong path if P contains only strong arcs. A fuzzy cut nodes w is a node in G whose removal from G reduces the strength of connectedness between some pair of nodes other than w . A fuzzy graph G is said to be a block if it is connected and has no cut nodes. If $\mu(u, v) > 0$, then u and v are called neighbours. If arc (u, v) is strong then v is called strong neighbor of u . The set of all neighbours of u is denoted by $N(u)$ and set of all strong neighbour of u is denoted by $N_s(u)$. A vertex u is a fuzzy end node if it has exactly one strong neighbor in G . An arc (u, v) is a fuzzy bridge if deletion of (u, v) reduces the strength of connectedness between some pair of vertices. In other words (u, v) is a fuzzy bridge if and only if there exist x, y such that (u, v) is an arc on every strongest x - y path. A vertex is a fuzzy cut vertex (f-cut vertex) of G if removal of it reduces the strength of connectedness between some other pair of vertices. Equivalently, x is a fuzzy cut vertex iff there exist u, v distinct from x such that x is on every strongest u - v path. A connected fuzzy graph $G:(V, \sigma, \mu)$ is a fuzzy tree (f-tree) if it has a spanning fuzzy sub graph $F:(V, \sigma, \nu)$, which is a tree where for all arcs (u, v) not in F there exist a path from u to v in F whose strength is more than $\mu(u, v)$. A maximum spanning tree of a connected fuzzy graph $G:(V, \sigma, \mu)$ is a fuzzy spanning subgraph $T:(V, \sigma, \nu)$ such that T^* is a tree and for which $\sum_{u \neq v} \nu(u, v)$ is maximum. A strong path P from u to v is uv geodesic if there is no shorter strong path from u to v and length of u - v geodesic is the geodesic distance between u to v denoted by $d_g(u, v)$. Let $G:(V, \sigma, \mu)$ be connected fuzzy graph and $P: u_0 - u_1 - \dots - u_n$ be any path in G . Then length of any path P is defined by as the sum of weights of the arcs in P and it is denoted by $L(P)$. If $n = 0$, define $L(P) = 0$ and for $n \geq 1$, $L(P) > 0$. Also, if G is disconnected then $L(P)$ may be Zero. For any two nodes u, v in G , the strong sum distance between u and

v is defined as $d_{ss}(u, v) = \text{Min}\{L(P) : P \text{ is strong } u\text{-}v \text{ path}\}$

3. FUZZY DETOUR SS-DISTANCE

Gary Chartand and Ping Zhang introduced the concept detour distance. This concept is extended to Fuzzy graph by J.P.Linda and M.S.Sunitha using g-disatnce. Fuzzy detour ss-distance is defined by E.EsaiArasi and S.Arulraj. In this section concepts of the fuzzy detour ss-distance, fuzzy detour ss-eccentricity, fuzzy detour ss-radius and fuzzy detour ss-diameter are discussed.

3.1 Definition

Let $G:(V, \sigma, \mu)$ be connected fuzzy graph and $P: u_0 - u_1 - \dots - u_n$ be any path in G . Then length of any path P is defined by $L(P) = \sum_{i=0}^{n-1} \mu(u_i, u_{i+1})$ if $n = 0$ define $L(P) = 0$ and for $n \geq 1$, $L(P) > 0$. Also if G is disconnected then $L(P)$ may be Zero. The detour ss-distance u and v is defined by $D_{ss}(u, v) = \text{Max}\{L(P) : P \text{ is any } u\text{-}v \text{ strong Path of } G\}$.

3.2 Example

Consider the fuzzy graph given in Fig 1

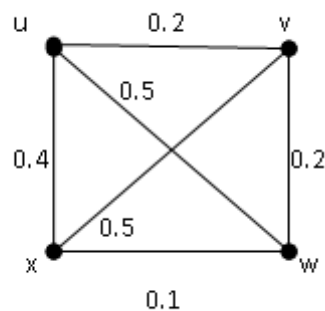


Fig.1

detour ss-distance are as follows. $D_{ss}(u, v) = 0.9$, $D_{ss}(u, w) = 1.1$, $D_{ss}(u, x) = 1.2$, $D_{ss}(v, w) = 1.4$, $D_{ss}(v, x) = 1.1$, $D_{ss}(w, x) = 1.2$.

Fuzzy detour ss-eccentricity $e_{D_{ss}}(u)$ of a vertex $u \in V$ is the fuzzy detour ss distance from u to a vertex farthest from u . Let u^* denote set of all Fuzzy detour ss eccentric vertex of u . Fuzzy detour ss radius of G , $r_{D_{ss}}(G)$ is the minimum fuzzy detour ss eccentricity among the vertices of G . Let $c_{D_{ss}}(G)$

be the collection of vertices of G such that its eccentricity is equal to $r_{D_{ss}}(G)$. A vertex u is a ss -central node of G if $e_{D_{ss}}(u) = r_{D_{ss}}(G)$. $c_{D_{ss}}(G)$ is the set of all central vertices of G . A Fuzzy detour ss diameter of G is the maximum eccentricity of the vertices and it is denoted by $d_{D_{ss}}(G)$. A vertex u is called Fuzzy detour ss - peripheral vertex if $e_{D_{ss}}(u) = d_{D_{ss}}(G)$.

3.3 Example

Consider fuzzy graph given in Fig 1.

Now $e_{D_{ss}}(u) = 1.2, e_{D_{ss}}(v) = 1.4, e_{D_{ss}}(w) = 1.4, e_{D_{ss}}(x) = 1.2$ and $r_{D_{ss}}(G) = 1.2, d_{D_{ss}}(G) = 1.4$

3.4 Definition

A fuzzy graph $G:(V, \sigma, \mu)$ is called fuzzy ss -detour graph if $D_{ss}(u, v) = d_{ss}(u, v)$ for every pair u and v of vertices of $G:(V, \sigma, \mu)$.

4.FUZZY DETOUR SS-PERIPHERY AND FUZZY DETOUR SS-ECCENTRIC SUBGRAPH

4.1 Definition

Let $G:(V, \sigma, \mu)$ be a fuzzy graph. The fuzzy subgraph of $G:(V, \sigma, \mu)$ induced by the fuzzy detour ss -Periphery of $G:(V, \sigma, \mu)$ denoted by $Per_{D_{ss}}(G)$.

4.2 Example

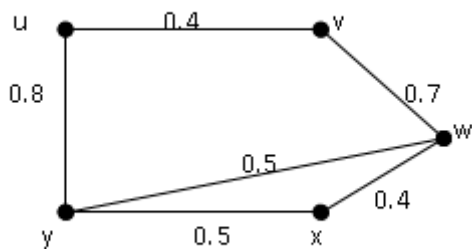


Fig 2

$D_{ss}(u, v) = 2, D_{ss}(u, w) = 1.3, D_{ss}(u, x) = 1.3, D_{ss}(u, y) = 0.8$
 $D_{ss}(v, w) = 0.7, D_{ss}(v, x) = 1.7, D_{ss}(v, y) = 1.2$
 $D_{ss}(w, y) = 0.5, D_{ss}(x, y) = 0.5, D_{ss}(w, x) = 1.$

Fuzzy detour ss -periphery of the graph given in Fig 3 is



4.3 Definition

A connected fuzzy graph $G:(V, \sigma, \mu)$ is a fuzzy detour ss -eccentric fuzzy graph if every node of $G:(V, \sigma, \mu)$ is a fuzzy detour ss -eccentric node. The fuzzy subgraph of $G:(V, \sigma, \mu)$ induced by the set of fuzzy detour ss -eccentric nodes of $G:(V, \sigma, \mu)$ is called the fuzzy detour ss -eccentric fuzzy graph of $G:(V, \sigma, \mu)$ denoted by $Ecc_{D_{ss}}(G)$.

4.4 Example

Consider the fuzzy graph given in Fig 2.

$u_{D_{ss}}^* = \{v\}, v_{D_{ss}}^* = \{u\}, w_{D_{ss}}^* = \{u\}, x_{D_{ss}}^* = \{v\}, y_{D_{ss}}^* = \{v\},$

Hence its ss -eccentric fuzzy sub graph is as given in Fig3.

Theorem 4.5

Let $G:(V, \sigma, \mu)$ be a connected fuzzy graph. Then u is a fuzzy detour ss -eccentric vertex iff u is fuzzy detour ss -peripheral vertex

Proof:

Let u be a fuzzy detour ss -eccentric vertex of $G:(V, \sigma, \mu)$. and let $u \in v_{D_{ss}}^*$. Suppose u is not a fuzzy detour ss -peripheral vertex. Let x and y be two fuzzy detour ss -peripheral vertices. so $D_{ss}(x, v) = d_{D_{ss}}(G)$. Let P be any x - y fuzzy detour in $G:(V, \sigma, \mu)$ and Q be any u - v fuzzy detour in $G:(V, \sigma, \mu)$

Case(i) Suppose u is not an end vertex, then there exist a path between x to u and y to v . so we can extend uv fuzzy detour to x or y . Which is a contradiction that u is a fuzzy detour ss -eccentric vertex of v . Thus u is a fuzzy detour ss -peripheral vertex.

Case(ii) Suppose u is an end vertex of $G:(V,\sigma,\mu)$. Let w be any vertex adjacent to u . Then w must belong to Q . Since G is connected, w is connected to some vertex say z of P . z may or may not lie on Q . In both cases the path from u to y or to x through w and z is larger than Q . Which contradicts that u is fuzzy detour peripheral node.

Conversely, assume u is a fuzzy detour ss-peripheral node of $G:(V,\sigma,\mu)$. Then there exist at least one more fuzzy detour ss-peripheral vertex v such that u is fuzzy detour ss-eccentric vertex of v .

5.FUZZY DETOUR SS-BOUNDARY VERTEX OF A FUZZY GRAPH

In this section, fuzzy detour ss-boundary vertex of a fuzzy graph is defined and some results connecting fuzzy detour ss-boundary vertex and complete vertex are obtained.

Definition 5.1

Let $G:(V,\sigma,\mu)$ be fuzzy connected graph. Then vertex v is said to be fuzzy detour ss-boundary vertex of a vertex u in $G:(V,\sigma,\mu)$ if $D_{ss}(u,v) \geq D_{ss}(u,w)$ for each neighbor w of v , while a vertex v is said to be fuzzy detour ss-boundary vertex of a fuzzy graph $G:(V,\sigma,\mu)$ if v is fuzzy detour ss vertex of some vertex of $G:(V,\sigma,\mu)$. Let $u_{D_{SS}}^b$ denote set of all fuzzy detour ss-boundary vertex of u .

Example 5.2

Consider the fuzzy graph given in Fig 4.

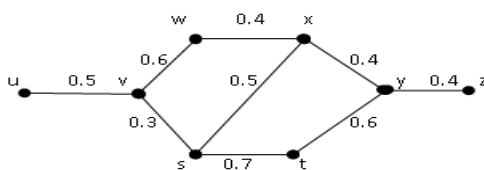


Fig 4.

$$u_{D_{SS}}^b = \{z\} \quad v_{D_{SS}}^b = \{u, z\}, \quad w_{D_{SS}}^b = \{u, z\}, \quad x_{D_{SS}}^b = \{u, z\}$$

Remark:5.3

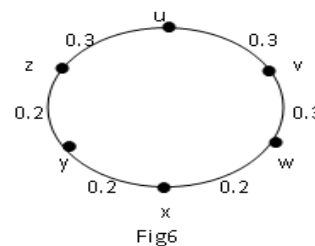
Vertex v of a crisp graph G is a boundary vertex of every vertex distinct from v iff v is a complete vertex of G . In fuzzy graph, based on fuzzy detour ss distance

a complete vertex need not be a detour ss-boundary vertex as shown figure

5. vertex w is a complete vertex but it is not a detour ss-boundary vertex of all other vertices. Also vertex u is a detour ss-boundary vertex of all other vertices but u is not a complete vertex.

Remark 5.4

In crisp graph, a vertex v is a boundary vertex iff v is not a cut vertex, but in fuzzy graph which is not a fuzzy tree, a fuzzy cut vertex can be a detour ss-boundary vertex as in Fig 6. In Fig 6 u is a detour ss-boundary vertex of v and u is fuzzy cut vertex.



Theorem 5.5

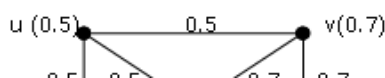
In a fuzzy graph $G:(V,\sigma,\mu)$ a vertex v is a cut vertex implies v is not a detour ss-boundary vertex

Proof:

Let v be a cut vertex of a fuzzy graph $G:(V,\sigma,\mu)$ and v be a detour ss-boundary vertex of u . Let G_1 be the component of $G-v$ which contains u and G_2 be any other component of $G-v$. If a node x is a neighbor of v that belongs to G_2 , then $D_{ss}(v,x) = D_{ss}(u,v) + y$ where $0 < y \leq 1$, which contradicts our assumption.

Theorem 5.6

Every eccentric vertex of a connected graph $G:(V,\sigma,\mu)$ is a boundary vertex.

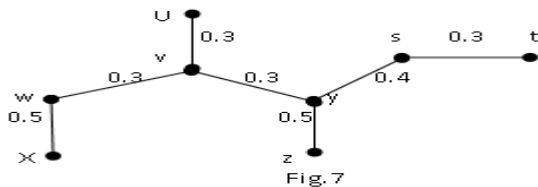


Proof:

Let $G:(V,\sigma,\mu)$ be connected fuzzy graph. Let v be a eccentric vertex of $G:(V,\sigma,\mu)$, then there exist a vertex u in V such that $D_{ss}(u,v) \geq D_{ss}(u,w)$ for every vertex w of V . Therefore $D_{ss}(u,v) \geq D_{ss}(u,w)$ for every vertex w of $N(v)$.

Remark 5.7

Converse of the above theorem need not be true. In Fig7, u is a boundary vertex of v but it is not an eccentric vertex.



Theorem 5.7

Let $G:(V,\sigma,\mu)$ be a fuzzy connected graph. Then $G:(V,\sigma,\mu)$ is fuzzy detour graph iff $G:(V,\sigma,\mu)$ is a fuzzy tree.

Proof:

Let $G:(V,\sigma,\mu)$ be fuzzy tree. Then there exist a unique strong path between any pair of vertices. Therefore $d_{ss}(u,v) = D_{ss}(u,v)$ for every pair of vertices of u and v .

Hence $G:(V,\sigma,\mu)$ is fuzzy detour graph.

Conversely, assume $G:(V,\sigma,\mu)$ is a fuzzy detour graph. So $d_{ss}(u,v) = D_{ss}(u,v)$ for every pair of vertices u and v of $G:(V,\sigma,\mu)$. If $|V|=2$ then the result is trivial and $G:(V,\sigma,\mu)$ is a fuzzy tree. Now let $|V| \geq 3$. Suppose $G:(V,\sigma,\mu)$ is not a fuzzy tree. Then there exist at least one pair of vertices u and v such that there exist more than one strong path from u to v . Let P and Q be two $u-v$ strong path. Then union of P and Q contains at least one cycle C in $G:(V,\sigma,\mu)$. Let x and y be two vertices in C such that $0 < \mu(x,y) \leq \mu(t,s)$ for all pair of vertices s and t in C . Then clearly $d_{ss}(x,y) \leq \mu(x,y) < (D_{ss}(x,y) \text{ in } C) \leq (D_{ss}(x,v) \text{ in } G)$. Which implies $d_{ss}(x,y) < D_{ss}(x,y)$ in G , Therefore $G:(V,\sigma,\mu)$ is not a fuzzy detour graph. Which is a contradiction. Therefore $G:(V,\sigma,\mu)$ is a fuzzy tree.

Theorem 5.8

In a fuzzy tree $G:(V,\sigma,\mu)$ no fuzzy cut vertex is a detour ss-boundary vertex of G .

Proof:

Let G be fuzzy tree. Suppose that there exist a cut vertex u such that u is a detour ss-boundary vertex of some vertex v of G . Let F be the unique maximum spanning tree of G . All internal vertices of F are fuzzy cut vertices of F . So u is an internal vertex of F . Detour ss-eccentric vertices of G and detour ss-eccentric vertices of F are all same. Since each detour ss-boundary vertices are detour sseccentric vertices of G and hence u is a fuzzy end vertex of F . which is a contradiction. Therefore no fuzzy cut vertex is a detour ss-boundary vertex of G .

Theorem 5.9

In a fuzzy tree $G:(V,\sigma,\mu)$, a detour ss-boundary vertex is a fuzzy end vertex.

Proof:

Let $G:(V,\sigma,\mu)$ be a fuzzy tree and v is a detour ss-boundary vertex of G . In a fuzzy tree every vertex is either a fuzzy cut vertex or a fuzzy end vertex. By theorem 5.8 no fuzzy cut vertex is a detour ss-boundary vertex of G . Hence u is a fuzzy end vertex of G .

6.FUZZY DETOUR SS-INTERIOR VERTEX OF A FUZZY GRAPH

In this section fuzzy detour ss-interior vertex and fuzzy detour ss-interior of a fuzzy graph are defined. A result connecting fuzzy detour ss-interior vertex and fuzzy detour ss-boundary vertex is obtained.

Definition 6.1

Any vertex y in a connected fuzzy graph $G:(V,\sigma,\mu)$ is said to lie between two other vertices say x and z (both different from y) with respect to fuzzy detour ss-distance if $D_{ss}(x,z) = D_{ss}(x,y) + D_{ss}(y,z)$.

Definition 6.2

A vertex v is fuzzy detour ss -interior vertex of a connected fuzzy graph $G:(V,\sigma,\mu)$ if for every vertex u distinct from v , there exist a vertex w such that v lies between u and w .

Definition 6.3

The fuzzy detour ss - interior of $G:(V,\sigma,\mu)$, $Int_{D_{ss}}(G)$ is the fuzzy sub graph of $G:(V,\sigma,\mu)$ induced by its fuzzy detour ss -interior vertices.

Example 6.4

For the fuzzy graph in Example 5.2 the detour ss -interior vertex are w, v, y, s are interior vertices .

Theorem 6.5

Let $G:(V,\sigma,\mu)$ be a fuzzy graph. A detour ss - boundary vertex of G is not a detour ss - interior vertex of G .

Proof

Let x be a detour ss -boundary vertex of a connected fuzzy graph $G:(V,\sigma,\mu)$. Suppose x is a detour ss -boundary vertex of a vertex y and x is a detour ss interior vertex of G . By definition of interior vertex, there exist a vertex u distinct from x and y such that x lies between u and y . Let P be that strong y - u bath in which v lies between u and w . ie, $p = y-x_1-x_2-\dots-x_j-x_{j+1}-\dots-x_k = u$ $1 < j < k$. Now $x_{j+1} \in N(x)$ and $D_{ss}(y, x_{j+1}) = D_{ss}(x, v) + k$. $0 < k \leq 1$, which contradicts that x is a boundary vertex of y .

7. CONCLUSION

In this paper, the concepts of fuzzy detour ss -boundary vertex and fuzzy detour ss -interior vertex in a fuzzy graph are introduced. Also some properties of fuzzy detour ss -boundary vertex, fuzzy detour ss -interior vertex and complete vertex are discussed.

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